Adapted list colouring of planar graphs

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Abstract

Given a (possibly improper) edge-colouring F of a graph G, a vertex colouring of G is *adapted to* F if no colour appears at the same time on an edge and on its two endpoints. If for some integer k, a graph G is such that given any list assignment L to the vertices of G, with $|L(v)| \geq k$ for all v, and any edge-colouring F of G, G admits a colouring c adapted to F where $c(v) \in L(v)$ for all v, then G is said to be *adaptably k-choosable*. In this note, we prove that K_5-minor-free graphs are adaptably 4-choosable, which implies that planar graphs are adaptably 4-colourable and answers a question of Hell and Zhu. We also prove that triangle-free planar graphs are adaptably 3-choosable and give negative results on planar graphs without 4-cycle, planar graphs without 5-cycle, and planar graphs without triangles at distance t, for any $t \geq 0$.

Keywords: Adapted colouring, list colouring, planar graphs.

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1 Introduction

The concept of adapted colouring of a graph was introduced by Hell and Zhu in [9], and has strong connections with matrix partition of graphs, graph homomorphisms, and full constraint satisfaction problems [4, 6, 7, 10]. The more general problem of adapted list colouring of hypergraphs was then considered by Kostochka and Zhu in [11], where an application to job assignment problems was also given.

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In this note, we study adapted list colourings of simple graphs. Let G be a simple graph (that is, without loops nor multiple edges), and let $F : E(G) \to \mathbb{N}$ be a (possibly improper) colouring of the edges of G. A k-colouring $c : V(G) \to \{1, \dots, k\}$ of the vertices of G is *adapted* to F if for every $uv \in E(G)$, $c(u) \neq c(v)$ or $c(v) \neq F(uv)$. In other words, the same colour never appears on an edge and both its endpoints. If there is an integer k such that for any edge colouring F of G, there exists a vertex k-colouring of G adapted to F, we say that G is *adaptably k-colourable*. The smallest k such that G is adaptably k-colourable is called the *adaptable chromatic number* of G, denoted by $\chi_{ad}(G)$.

Note that in [9] and [11], the authors require that the edge colouring F is a k-colouring. Even though we enable F to take any integer value, it is easy to see that our definition is equivalent to the original definition (whereas its extension to adapted list colouring is more natural). Let $L : V(G) \to 2^{\mathbb{N}}$ be a list assignment to the vertices of a graph G, and F be a (possibly improper) edge colouring of G. We say that a colouring c of G adapted to F is an *L-colouring adapted to F* if for any vertex $v \in V(G)$, we have $c(v) \in L(v)$. If for any edge colouring F of G and any list assignment L with $|L(v)| \geq k$ for all $v \in V(G)$ there exists an L-colouring of G adapted to F, we say that G is *adaptably k-choosable*. The smallest k such that G is adaptably k-choosable is called the *adaptable choice number* of G, denoted by $\mathrm{ch}_{ad}(G)$.

Since a proper vertex k-colouring of a graph G is adapted to any edge colouring of G, we clearly have $\chi_{ad}(G) \leq \chi(G)$ and $\mathrm{ch}_{ad}(G) \leq \mathrm{ch}(G)$ for any graph G, where $\chi(G)$ is the usual chromatic number of G, and $\mathrm{ch}(G)$ is the usual choice number of G. Using the Four-Colour Theorem and a theorem of Thomassen [13], this proves that for any planar graph G, $\chi_{ad}(G) \leq 4$ and $\mathrm{ch}_{ad}(G) \leq 5$. In [9], Hell and Zhu proved that there exist planar graphs that are not adaptably 3-colourable, and asked whether it would be possible to prove that every planar graph is adaptably 4-colourable without using the Four-Colour Theorem.

A graph H is called a *minor* of G if a copy of H can be obtained by contracting edges and/or deleting vertices and edges of G. A graph is said to be *H-minor-free* if it does not have H as a minor. Planar graphs are known to be a proper subclass of K_5-minor-free graphs. In this note, we answer to the question of Hell and Zhu by proving the following stronger statement:

Theorem 1 *Every K_5-minor-free graph is adaptably 4-choosable.*

Observe that this does not hold for the usual list colouring, since Voigt [15] proved that there exist planar graphs which are not 4-choosable.

Triangle-free planar graphs are known to be 3-colourable [5, 14] and 4-choosable (it is easy to prove that they are 3-degenerate using Euler Formula). On the other hand Voigt [16] proved that there exist triangle-free planar graphs that are not 3-choosable. In Section 3, we prove the following theorem:

Theorem 2 *Every triangle-free planar graph is adaptably* 3*-choosable.*

In Section 4, we investigate a problem related to a question of Havel [8]. We prove that for all t, there exist planar graph without triangles at distance less than t, which are not adaptably 3-choosable. In Sections 5 and 6, we prove that there exist planar graphs without 4-cycles, and planar graph without 5-cycles, which are not adaptably 3-colourable. These negative results seem to indicate that it may be hard to have a weaker hypothesis in Theorem 2.

2 K_5-minor-free graphs

Theorem 1 is a consequence of Lemma 2.3 in this section. Note that the adaptable 4-choosability of planar graphs can be deduced directly from Lemma 2.1.

Lemma 2.1 *Let G be an edge-coloured plane graph, and let $C = (v_1, \ldots, v_k)$ be its outer face. Let ϕ be an adapted colouring of v_1 and v_2. Suppose finally that any vertex $v \in C$ distinct from v_1 and v_2 has a colour list $L(v)$ of size at least three and every vertex $v \in V(G) \setminus C$ has a colour list $L(v)$ of size at least four. Then the colouring ϕ can be extended to an adapted L-colouring of G.*

Proof. We prove this lemma by induction on $|V(G)|$. If $|V(G)| = 3$, the assertion is trivial. Suppose now that $|V(G)| \geq 4$ and assume that the assertion is true for any smaller graphs.

Since the subgraph G_C of G induced by C is an outerplanar graph, it contains two vertices v_i and v_j of degree at most two which are not adjacent in G_C and which are not cut-vertices of G_C. These vertices v_i and v_j are neither cut-vertices of G nor incident to a chord of C, and one of them (say v_i), is distinct from v_1 and v_2. Let $\alpha \in L(v_i)$ be a colour distinct from the colours of the edges v_iv_{i+1}, v_iv_{i-1}. For each neighbour x of v_i not in C, we remove the colour α from the colour list of x. Applying the induction hypothesis to $G \setminus v_i$ and then colouring v_i with α yields an adapted list colouring of G. ∎

Lemma 2.2 *Let G be an edge-coloured plane graph. Suppose that every vertex v of G has a list $L(v)$ of size at least four. Let H be a subgraph of G isomorphic to K_2 or*

K_3, and let ϕ be an adapted L-colouring of H. Then ϕ can be extended to an adapted L-colouring of G.

Proof. Let G be a counterexample with minimum order. If H is isomorphic to K_2, then consider a face incident to H as the outer face and apply Lemma 2.1 to this planar embedding of G.

Assume now that H is isomorphic to K_3 and $V(H) = \{u, v, w\}$. If H is a separating 3-cycle, then let G_1 (resp. G_2) be the graph induced by the vertices of H and the vertices inside (resp. outside) of H. By the minimality of G, extending ϕ to G_1 and to G_2 yields an adapted L-colouring of G. Suppose now that H is not a separating 3-cycle, and assume that H bounds the outer face of G. Let $G' = G \setminus w$ and let L' be the list assignment defined by $L'(x) = L(x) \setminus \{\phi(w)\}$ for every vertex x adjacent to w (and distinct from u, v) and by $L'(x) = L(x)$ for any other vertex distinct from u and v. Lemma 2.1 applied to G' allows to extend ϕ to G. ∎

Lemma 2.3 *Let G be an edge maximal K_5-minor-free graph. Suppose that every vertex v of G has a list $L(v)$ of size at least four. Let H be a subgraph of G isomorphic to K_2 or K_3, and let ϕ be an adapted L-colouring of H. Then ϕ can be extended to an adapted L-colouring of G.*

Proof. Let G be a counterexample with minimum order. Then G is not isomorphic to the Wagner graph (which is 3-regular, and hence adaptably L-colourable given a precolouring of H), and by Lemma 2.2, G is not a planar triangulation. It follows from Wagner's theorem [17], that $G = G_1 \cup G_2$ where G_1, G_2 are proper subgraphs of G such that $G_1 \cap G_2$ is isomorphic to K_2 or K_3. Clearly, $H \subseteq G_1$ or $H \subseteq G_2$. Without loss of generality, assume that $H \subseteq G_1$. By minimality of G, we can extend ϕ to G_1. This gives an adapted colouring to $G_1 \cap G_2$ which can be extended to G_2, by the minimality of G. This yields an extension of ϕ to an adapted L-colouring of G. ∎

3 Triangle-free planar graphs

Theorem 2 is a consequence of the following theorem:

Theorem 3 *Suppose G is an edge-coloured simple triangle-free plane graph, $C = (v_1, v_2, \cdots, v_k)$ is the outer face. Suppose L is a list assignment that assigns to each vertex x a set $L(x)$ of 3 permissible colours, except that some vertices on C have only 2 permissible colours. However, each edge of G has at least one end vertex x which has 3 permissible colours. Then G is adaptably L-colourable.*

Proof. We may assume G is connected and prove the theorem by induction on the number of vertices. If $|V(G)| \le 4$, then the theorem is obviously true.

Assume $|V(G)| \ge 5$. A path $P = (v_i, x, v_j)$ is called a *long chord* of C connecting v_i and v_j, if $v_i, v_j \in C$, $x \notin C$ and $|L(v_i)| + |L(v_j)| = 5$. Let $\mathcal{P}$ be the set of chords, long chords, and cut-vertices of C. Suppose $P \in \mathcal{P}$ is a chord (v_i, v_j) or a long chord (v_i, x, v_j) connecting v_i and v_j. We denote by A_P and B_P the two components of $C - \{v_i, v_j\}$, and assume that $|A_P| \le |B_P|$. If $P \in \mathcal{P}$ is a cut-vertex of C, we denote by A_P the smallest component of $C - P$. Let $P^* \in \mathcal{P}$ be a chord, long chord, or cut-vertex, for which $|A_{P^*}|$ is minimum.

Claim A_{P^*} contains a vertex v_t which is not a cut-vertex, such that $|L(v_t)| = 3$ and v_t is not contained in any chord or long chord of C.

First observe that A_{P^*} does not contain any cut-vertex, since otherwise this would contradict the minimality of P^*. Assume that P^* is a cut-vertex v. Then A_{P^*} contains at least two adjacent vertices v_i and v_{i+1}, and both of them are neither contained in a chord nor in a long chord of C by the minimality of P^*. By the hypothesis, there is a $t \in \{i, i+1\}$ such that $|L(v_t)| = 3$.

Assume $P^* = (v_i, x, v_j)$ is a long chord, $|L(v_j)| = 2$ and $A_{P^*} = (v_{i+1}, v_{i+2}, \cdots, v_{j-1})$. Then $|L(v_{j-1})| = 3$, for otherwise $v_j v_{j-1}$ is an edge of G connecting two vertices each with 2 permissible colours, in contrary to our assumption. Since G is triangle-free, v_{j-1} is not adjacent to x. If v_{j-1} is contained in a chord or a long chord P', then we would have $A_{P'} \subset A_{P^*}$ and hence $|A_{P'}| < |A_{P^*}|$, in contrary to our choice of P^*.

Assume $P^* = (v_i, v_j)$ is a chord, and $A_{P^*} = (v_{i+1}, v_{i+2}, \cdots, v_{j-1})$. Since G is triangle-free, $v_{i+1} \ne v_{j-1}$. Since each edge of G has at least one end vertex x which has 3 permissible colours, there exists $t \in \{i+1, i+2\}$ such that $|L(v_t)| = 3$. By the same argument as above, v_t is not contained in any chord or long chord of C. This completes the proof of the claim.

Let $v_t \in C$ be a vertex which is not a cut-vertex, such that $|L(v_t)| = 3$ and v_t is not contained in any chord or long chord of C. Let $\alpha \in L(v_t)$ be a colour distinct from the colours of the two edges $v_{t-1}v_t$ and $v_t v_{t+1}$. Let $G' = G - v_t$ and let L' be a list assignment of G' defined as $L'(x) = L(x) - \{\alpha\}$ if x is a neighbour of v_t distinct from v_{t-1}, v_{t+1}, and $L'(x) = L(x)$ otherwise. Then $L'(x)$ contains 3 colours for each interior vertex x of G' and $L'(x)$ contains at least 2 colours for each vertex x on the outer face of G', since v_t is not contained in any chord of C. Moreover, since v_t is not contained in any long chord of C, it follows that each edge of G' has at least one end vertex x which has 3 permissible colours. By induction hypothesis, G' is adaptably L'-colourable. Any L'-colouring of G' can be extended to an L-colouring of G by colouring v_t with colour α. So G is adaptably L-colourable. ∎

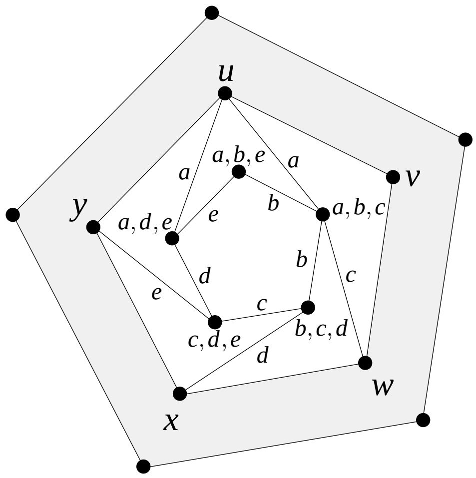

Figure 1: The construction of H_k.

4 Planar graphs without triangles at distance k

The distance between two triangles xyz and uvw is the minimum distance between a vertex of $\{x, y, z\}$ and a vertex of $\{u, v, w\}$. For any graph G, we denote by $d_t(G)$ the minimum distance between two triangles of G. If G contains at most one triangle, we take $d_t(G)$ to be infinite. Havel [8] asked the following question: is it true that for some k, every planar graph G with $d_t(G) \geq k$ is 3-colourable? Havel showed that such an integer k is at least 2, disproving a conjecture of Grűnbaum. In [1], Aksionov and L.S Mel'nikok proved that such a k is at least 4, and conjectured that the real value should be 5.

Since triangle-free planar graphs are adaptably 3-choosable, it is interesting to see if anything can be said about a relaxation similar to Havel's problem : is there an integer k, such that any planar graph G with $d_t(G) \geq k$ is adaptably 3-choosable? In the following, we prove that such a k does not exist: more precisely, for every k we construct a planar graph where every two triangles are at distance at least $2k$ apart, which is not adaptably 3-choosable.

Let us define the distance between two faces $\mathcal{F}_1$ and $\mathcal{F}_2$ of a graph as the minimum distance between a vertex of $\mathcal{F}_1$ and a vertex of $\mathcal{F}_2$. A face containing exactly k vertices is called a k-face. In the following, we construct inductively the plane graph H_i, such that the following is verified at each step:

(a) H_i is triangle-free.

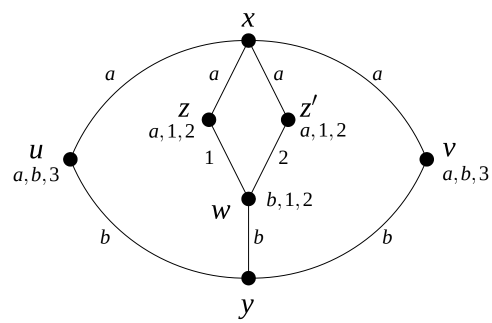

Figure 2: $H(a, b)$.

(b) H_i contains exactly two 5-faces (the outer face and another face, say $\mathcal{F}_i$). Moreover, the distance between these two faces is exactly i.

(c) Assume that the outer face is coloured with five distinct colours a, b, c, d and e in clockwise order. Then there exist an edge-colouring F_i of H_i and a list assignment L_i with $|L_i(v)| = 3$ for every vertex v which is not incident to the outer face, such that H_i has a unique L_i-colouring adapted to F_i. Moreover, this colouring is such that $\mathcal{F}_i$ is coloured with a, b, c, d and e in clockwise order.

Let H_0 be a 5-cycle. Then the three properties are trivially verified. Assume that for some $i \geq 1$, H_{i-1} also verifies these properties. Fix five different colours a, b, c, d, and e (in clockwise order) on the vertices of the outer face of H_{i-1}. By property (3), there exist an edge-colouring F_{i-1} of H_{i-1} and a list assignment L_{i-1} with lists of size three, such that H_{i-1} has a unique L_{i-1}-colouring adapted to F_{i-1}. In this colouring, the vertices u, v, w, x, and y of the 5-face $\mathcal{F}_{i-1}$ are coloured with a, b, c, d and e respectively. Let H_i be the graph obtained from H_{i-1} by adding five new vertices inside $\mathcal{F}_{i-1}$, as depicted in Figure 1. This figure also shows how to extend F_{i-1} and L_{i-1} to an edge-colouring F_i and a list-assignment L_i of H_i.

Since u and w are coloured with a and c respectively, the new vertex v' adjacent to u and w must be coloured with b. The new vertex w' adjacent to v' and x must be coloured with c; the new vertex x' adjacent to w' and y must be coloured with d; the new vertex y' adjacent to x' and y must be coloured with e, and the new vertex u' adjacent to y' and v' must be coloured with a. The graph H_i is still triangle-free, and only contains two 5-faces: the outer face and $\mathcal{F}_i = u'v'w'x'y'$. Moreover these two faces are at distance exactly $i-1+1 = i$. Hence, the graph H_i verifies properties (a), (b), and (c). We denote by G_i the graph obtained from H_i by adding inside the face $\mathcal{F}_i$ a 3-vertex z adjacent to u', w', and x'. We give the edges zu', zw' and zx' colours a, c, and d respectively, and we assign the list $\{a, c, d\}$ to z. Observe that the graph G_i contains only one triangle (which is at distance i from the outer face), and that the colouring of the outer face cannot be extended to an adapted list-colouring of G_i.

Let $H(a, b)$ be the edge-coloured graph depicted in Figure 2. Assume that x and y are coloured with a and b respectively. Then u and v must be coloured with 3, and w must be coloured either 1 or 2. If it is coloured with 1, the 5-face $xzwyu$ has its vertices coloured with a, 2, 1, b and 3. Otherwise, the 5-face $xvywz'$ has its vertices coloured with a, 3, b, 2, 1. Let $G(a, b)$ be the graph obtained from $H(a, b)$ by plugging the widget G_k in each of the two 5-faces (that is, each of these two faces becomes the outer face of a graph G_k). Using what has been done before, we know that with a suitable edge-colouring of the two widgets, there exists a list assignment with lists of size three, such that the colouring of $H(a, b)$ cannot be extended to a colouring of $G(a, b)$. Hence, if x and y are coloured with a and b respectively, this cannot be extended to an adapted list colouring of $G(a, b)$.

Consider 9 copies of $G(a, b)$, with $(a, b) \in \{4, 5, 6\} \times \{7, 8, 9\}$, and identify all the vertices x (resp. y) of these copies into a single vertex x^* (resp y^*). Assign the colour lists $\{4, 5, 6\}$ and $\{7, 8, 9\}$ to x^* and y^* respectively. Assume that there exists an adapted list colouring f of this graph, then there exist no adapted list colouring of the copy of $G(f(x^*), f(y^*))$, which is a contradiction. Hence, this planar graph is not adaptably 3-choosable, and any two triangles are at distance at least $2k$ apart.

5 Planar graphs without 4-cycles

In this section, we prove that there exist planar graphs without 4-cycles, which are not adaptably 3-colourable. Let $H(a, b, c)$ be the edge-coloured graph depicted in Figure 3. Consider that $\{a, b, c\} = \{1, 2, 3\}$, and assume that the vertices u and v of $H(a, b, c)$ are coloured with a and b respectively. Then at least one of the vertices w and w' is coloured with c. By symmetry, we can assume that w is coloured with c. Then x must be coloured with a, y must be coloured with c, and z and z' must be coloured with b. It is easy to check that in this situation, the remaining subgraph induced the vertices at distance one or two from z and z' cannot be adaptably coloured. Hence, if u and v are coloured with a and b, this colouring cannot be extended to an adapted 3-colouring of $H(a, b, c)$.

For every $1 \leq a \leq 3$, let b and c be the two colours from $\{1, 2, 3\}$ distinct from a. We denote by G_a the edge-coloured graph obtained from $H(a, b, c)$ and $H(a, c, b)$ by contracting the two vertices u (resp. v) into a single vertex u^* (resp.v^*). Observe that in any adapted 3-colouring of G_a, if u^* is coloured with a then v^* is also coloured with a.

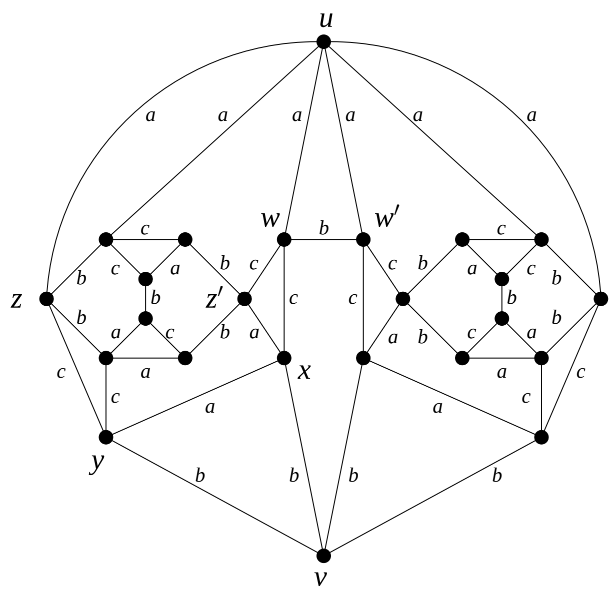

Figure 3: $H(a, b, c)$.

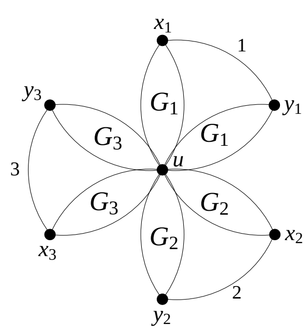

Figure 4: A planar graph without 4-cycle, which is not adaptably 3-colourable.

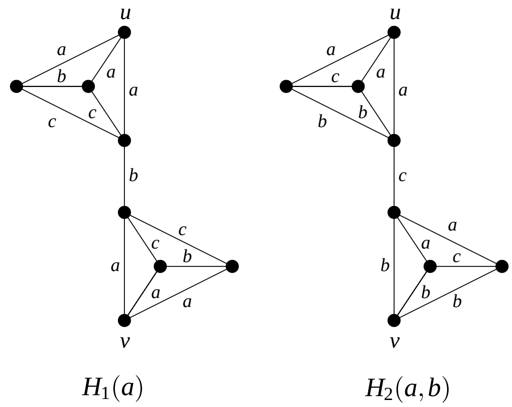

Figure 5: $H_1(a)$ and $H_2(a,b)$.

Consider now an adapted 3-colouring of the construction of Figure 4, which does not contain any 4-cycle. If the vertex u is coloured with $1 \leq i \leq 3$, then the two vertices x_i and y_i are both coloured with i, which is a contradiction since they are linked by an edge coloured with i. Hence, this graph is not adaptably 3-colourable.

6 Planar graphs without 5-cycles

In this section, we prove that there exist planar graphs without 5-cycles, which are not adaptably 3-colourable. For any $\{a,b,c\} = \{1,2,3\}$, let $H_1(a)$ and $H_2(a,b)$ be the two C_5-free planar graphs depicted in Figure 5. It is easy to check that in $H_1(a)$, if the vertices u and v are coloured with a, then this colouring cannot be extended to an adapted colouring of $H_1(a)$. Similarly in $H_2(a,b)$, if u and v are coloured respectively with a and b ($a \neq b$), then this colouring cannot be extended to an adapted colouring of $H_2(a,b)$.

Consider the three graphs $H_1(a)$ for $1 \leq a \leq 3$, and the six graphs $H_2(a,b)$ with $1 \leq a \neq b \leq 3$. Contract the nine vertices u (resp. v) of these graphs into a single vertex u^* (resp. v^*). Assume that there exists an adapted 3-colouring f of this graph. If $f(u^*) = f(v^*)$ then the copy of $H_1(f(u^*))$ is not adaptably 3-colourable, which is a contradiction. Otherwise $f(u^*) \neq f(v^*)$ and the copy of $H_2(f(u^*), f(v^*))$ is not adaptably 3-colourable, which is also a contradiction. Hence, this graph is planar and without 5-cycles, but is not adaptably 3-colourable.

7 Conclusion

In this note, we proved that triangle-free planar graphs are adaptably 3-choosable, whereas C_4-free planar graphs and C_5-free planar graphs are not even adaptably 3-colourable. We also showed that for any $k \geq 0$, there exist planar graphs without triangles at distance k which are not adaptably 3-choosable. However, the question remains open for adapted colouring:

Question 7.1 *Is there an integer k, such that every planar graph G with $d_t(G) \geq k$ is adaptably 3-colourable?*

If the answer to this question is negative, it implies that the answer to the original problem of Havel is also negative, whereas a positive answer to the original problem of Havel would imply a positive answer to Question 7.1.

In 1976, Steinberg conjectured that planar graphs without cycles of length 4 and 5 are 3-colourable (see [12] for a survey). We can ask the same for adapted 3-colouring and adapted 3-choosability :

Question 7.2 *Are planar graphs without 4-cycles and 5-cycles adaptably 3-colourable?*

Question 7.3 *Are planar graphs without 4-cycles and 5-cycles adaptably 3-choosable?*

A weaker version of the problem of Steinberg was proposed by Erdős in 1991: he asked what is the smallest i, such that every planar graph without cycles of length 4 to i is 3-colourable? The same can be asked for adapted 3-colouring and adapted 3-choosability:

Question 7.4 *What is the smallest i, such that every planar graph without cycles of length 4 to i is adaptably 3-colourable?*

Question 7.5 *What is the smallest i, such that every planar graph without cycles of length 4 to i is adaptably 3-choosable?*

Note that by [3], the answer of Question 7.4 is at most 7, and by [2, 18], the answer of Question 7.5 is at most 9.

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